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Designing worked examples for learning tangent lines to circles

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Abstract. Geometry is a branch of mathematics that deals with shape and space, including the circle. A difficult topic in the circle may be the tangent line to circle. This is considered a complex material since students have to simultaneously apply several principles to solve the problems, these are the property of circle, definition of the tangent, measurement and Pythagorean theorem. This paper discusses designs of worked examples for learning tangent line to circles and how to apply this design to an effective and efficient instructional activity. When students do not have sufficient prior knowledge, solving tangent problems might be clumsy, and as a consequence, the problem-solving activity hinders learning. According to a Cognitive Load Theory, learning occurs when students can construct new knowledge based on the relevant knowledge previously learned. When the relevant knowledge is unavailable, providing students with the worked example is suggested. Worked example may reduce unproductive process during learning that causes extraneous cognitive load. Nevertheless, worked examples must be created in such a way facilitate learning.

1. Introduction

Problem-solving is the core activity in mathematics learning [1]. The National Council of Teachers of Mathematics (NCTM) emphasizes that it is the great disadvantage if students have knowledge of mathematics but are unable to use it to solve problems [2]. It is understood that learning by problem-solving means that students are given a problem solving then asked to think mathematically to discover the underlying knowledge either by themselves or by discussing in a small group [3-6].

For novice learners, however, cognitive load theorists suggest that without sufficient knowledge base, they will have difficulty to discover the new knowledge [1, 7]. For example, a novice student is given a stimulating mathematics problem: *Two wheels are 1,5 meters apart. A real-world is set around these two wheels. How long is the rope if the radii of the wheels are 40 cm and 60 cm respectively*. A novice may not know whether this mathematics problem is about the length of tangent lines to the circles, and to solve this, the Pythagorean theorem is required. They will search the solution by repeatedly trying until satisfying, unless for those who are giving up. Arguably, this activity might be unlikely directed to learning. Therefore, novice learners require assistance or scaffolding to build a knowledge base relevant to be applied in the problem-solving task to create a productive learning environment for them.

When teaching mathematics to a class of students, it is common to show students a worked example of how a problem to be solved. Unfortunately, showing a worked example only does not always successfully attract student's attention to acquiring the knowledge. It is agreed that student who is responsible for constructing the knowledge [1] based on their previous knowledge [2, 8]. To learn the worked example, the student must be able to understand the problem statement and also how the problem to be solved as well as why the strategy is applied, as provided in the worked example [9]. Consequently, one worked example only may not be sufficient. Therefore, if the worked example is preferred as a learning strategy, instructors must be able to design the worked example in such away it assists students to understand how the problem is solved and at the same time it is motivating to be learned.

According to a cognitive load theory [9-11], worked example as a learning strategy can increase the effectiveness of studying problem solving particularly for novice students. Atkinson [7] suggests that a worked example should provide clear explanations. Such explanations will guide students to understand the problem and how it is solved by following the step-by-step the solution. Thus, when creating the example, the instructor should note that students who learn the worked example may have lack of prior knowledge relevant to the to-be-learned problem-solving. The instructor should be able to anticipate how the student will grasp the explained in such a way that the explanation could assist their learning.

Sweller, Ayres and Kalyuga [11] describes the associated effects with the implementation of worked examples; these are (1) the expertise reversal effect, (2) the split-attention effect and the redundancy effect, and (3) the pairing effect (pairs of worked example and problem-solving). Firstly, the expertise reversal effect explains that the level of prior knowledge possessed by students may determine the effectiveness of instruction. Accordingly, the worked example instruction is suitable for novices since it provides explicit instruction. Secondly, the worked examples should be formatted in such a way it does not cause extraneous cognitive load [12]. The extraneous cognitive load may be triggered by how the information is presented. This type of cognitive load does not direct students to construct new knowledge systematically. Thirdly, to motivate students to automate their just-learned knowledge from the worked example, a pair of the isomorphic problem may be provided after each example [13]. When students are solving this paired problem without necessarily looking at the example of a solution, they are instructed to recall the knowledge they construct based on the worked example. They may develop their strategy of solving the problem based on their understanding of the worked example.

Creating worked example for specific learning material, such as geometry, might require some considerations of the cognitive load principles. Geometry is the branch of mathematics that has been part of school mathematics for long. It is studied by solving problems that commonly consist of visualization [2]. As discussed above, for novice students, learning geometry by problem-solving may be challenging. Hence, facilitating them with worked examples would be suggested. The challenge of creating worked example for geometry might be in showing the visualization and in formatting the pictures and explanation without escalating the extraneous cognitive load.

The aim of this paper is to discuss the design of worked example for learning a topic in Geometry, namely tangent lines to circle. Although basic properties of the circle are introduced at elementary school, tangent lines to the circle are studied at intermediate grades [2]. This topic is favored since it encourages students to apply simultaneous knowledge on geometry and algebra, as well as it has some interesting applications in real-world contexts. In fact, problem-solving in this topic is not easy to accomplish for novices.

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2. Methods

2.1. The design of the worked example for learning tangent lines to circle

The problem presented in the second paragraph on the first page of this paper is an example of the problem of the tangent line to circle in real life context. There are two types of tangent lines to circle. The hint to solve the such problem is to create a right-angled triangle where the sides are a line that is parallel to the tangent line, the central line and the radii on the tangent point. The parallel line must be drawn. Otherwise, there are two right-angle triangles which make the problem solving more difficult.

The design of the worked example should follow the principles derived from the cognitive load theory. First of all, the expertise reversal principle mentioned the worked example is studied by novice learners. These are students who do not possess sufficient prior knowledge. The required prior knowledge includes circle, the definition of a tangent line, properties of the tangent line to a circle, parallelism, and Pythagorean theorem. The next principle concerns with how to lay the visualization of the problem and the problem solution steps. To avoid the split attention effect, an integrated format should be followed. This may be done by integrating the solution steps inside the picture. To assist students finding relevant information that needs to be integrated from both sources (picture and text), the same color may be applied. Therefore, several colors will perform the worked example. Moreover, the redundancy effect may be minimized by eliminating the explanatory text for the self-contained picture. Last but not the least is to provide an isomorphic problem to solve by the student themselves after each example. By pairing a worked example with a similar problem, students will have the opportunity to automate what they have just learned from the example. The instruction, the worked example, and the paired problem can be described as following.

***The instruction*:** Study the worked example until you understand how the problem is solved, then try to solve the paired problem by yourself without looking the example.

***The worked example 1*:** AB is the tangent line to circles Q and circle R shown in Figure 1. Evaluate the length of the tangent line AB!

Because of parallelism, AB = PR and, hence for triangle PQR:

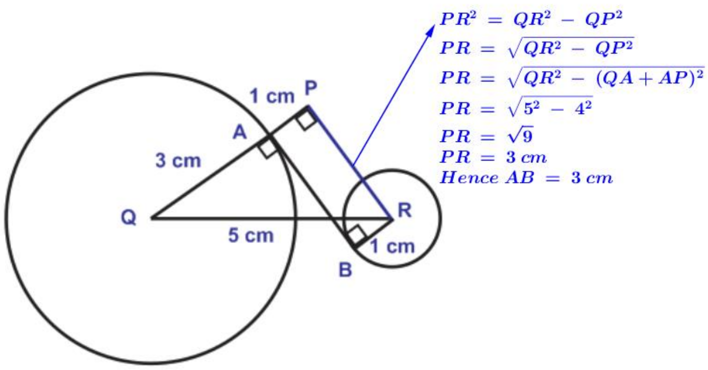


Figure 1. Example 1

The paired problem 1. Evaluate the length of the tangent line to circle Q and circle R as shown in the Figure 2 (not to scale). Given QR = 20.

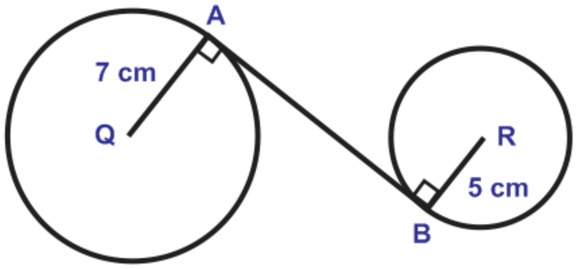


Figure 2. Example 2

The worked example 2. AB is the tangent line to circle Q and circle R shown in Figure 3. Since AB = PR, and triangle PQR has the right angle on point P, and the shortest distance between both triangles is ST, as well that points Q, S, T and R collinear, hence:

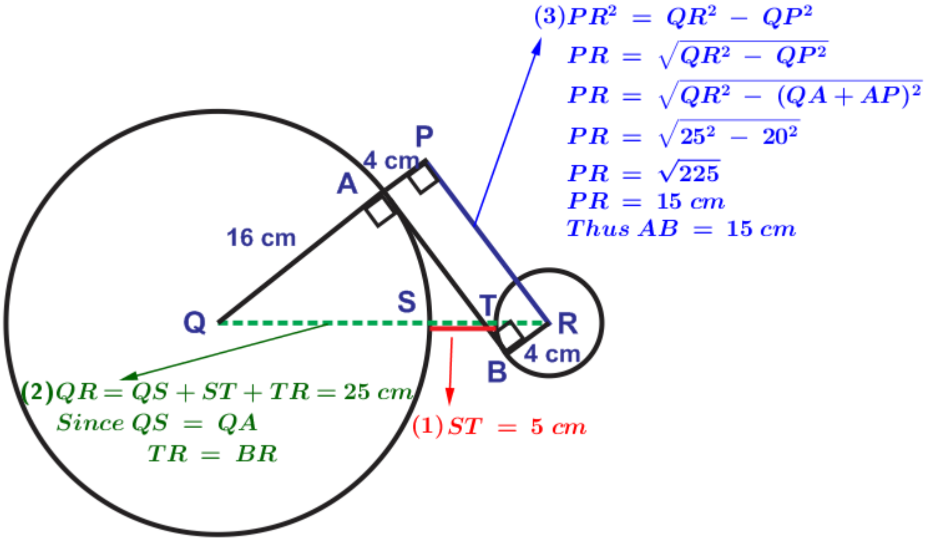
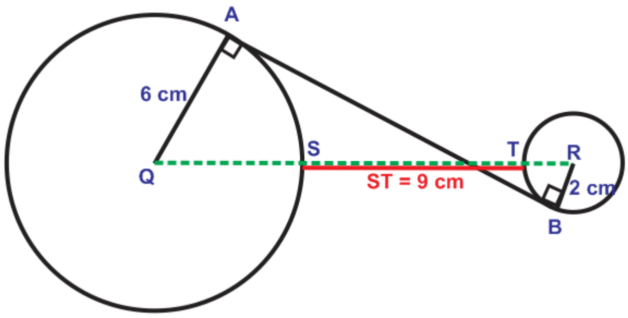


Figure 3. Example 3

The paired problem 2. The shortest distance between both circles is shown by ST thus Q, S, T, and R are collinear. Evaluate the length of tangent line to circle Q and circle R as shown in the Figure 4.



(not to scale).
Figure 4: the paired problem 2

The paired problem is created to be similar to the example. Arguably, this similarity would assist students to practice their new knowledge. Too much difference could probably confuse students. Furthermore, there should be several pairs of example and problem. The pairs should have a variety of context, hence broaden learning.

3. Result and Discussion

3.1 The implementation of the worked example

Evidence on the effectiveness of worked example based instruction has been shown by cognitive load theorists [9, 11, 14, 15] using control experiments. How to implement the instruction in the classroom may consider the nature of the interaction between teacher and students in the classroom, which may be different with the experiment procedure used by the researchers. As explained by the expertise reversal effect, the worked example instruction is applicable for novices. Therefore, the teacher should ensure that the learning material is complex and new problem solving for the students.

In the classroom, the teacher should remind students the pre-requisite knowledge before students use the worksheet containing the worked example pairs. During the instruction, it is also important to motivate students to understand meaningfully rather than just memorizing. For a complex learning material, individually acquiring the problem and the solution steps are suggested than with companion of their peers [14, 15]. After finishing all of the paired problem solving, students may be given the correct answer for them clarifying their work result. Such clarification is important to avoid students from misunderstanding. Besides, it is motivating since students would feel satisfied with their learning achievement. For an alternative, the instructor may ask the students to explain how they understand how to solve the tangent problems. Explaining is also a suggested strategy to clarify and deepen the acquired knowledge.

4. Conclusion

Learning geometry, particularly the tangent lines to circles, for novice students can be facilitated by worked examples. Based on cognitive load theory, the worked example based instruction should follow at least three principles to minimise extraneous cognitive load. These are (1) considering the expertise reversal effect by understanding whether students already possess the prior knowledge, (2) avoiding the split-attention effect and the redundancy effect, by considering how to lay the picture and texts and how to apply colours in the visualisation, and (3) providing pairs of worked example and problem solving with the final answer to clarify the student's result of problem solving. The worked example for learning the tangent lines to circles could be designed as an integrated format. In this format, the step-by-step solution is presented in the picture of the circles, and each step is written in the same color as the corresponding part of the picture. During the classroom implementation, the teacher may improve the effectiveness by motivating students to study meaningfully. To obtain empirical evidence for the effectiveness of the design, the future investigation is needed such as by comparing this design with the others.

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